FLST WS 2008/2009 – Semantics – Exercise Sheet 2

Manfred Pinkal

1. Are the following formulae logically valid, contradictory, or contingent?

   a. $\vDash \exists x \forall y(R(x,y) \leftrightarrow \neg R(y,y))$?
   b. $\vDash \exists x \forall y R(x,y) \rightarrow \forall y \exists x R(x,y)$?
   c. $\vDash \forall y \exists x R(x,y) \rightarrow \exists x \forall y R(x,y)$?

For all of the following exercises, assume that constants have the following types:

j, x, y : e
M, Y: <e,t>
S: <<e,t>,<e,t>>
C: <<e,t>,t>
R: <e,<e,t>>

2. Which of the following expressions are well-formed expression of type-theory?

   a. j(M)
   b. S(M(j))
   c. S(M)
   d. (S(M))(j)
   e. C(M)
   f. (C(M))(j)

3. Determine the types of A and B. The complete expression should be of type t.

   a. (A(M))(j)
   b. A(M(j))
   c. (S(M))(A)
   d. (S(M))(j)
   e. B((S(M))(A))

4. Are the following expressions well-formed? If yes, what is the type of the complete expression?

   a. $\lambda x(M(x))(C)$
   b. $\lambda x(M(x))(j)$
   c. $S(\lambda x(M(x)))$
   d. $\lambda Y(Y(j))(M)$
   e. $\lambda x \lambda Y(Y(x))$
   f. $\lambda x(M(x)) \wedge M(j)$
   g. $\lambda Y((S(\lambda x(M(x))))(j) \wedge C(Y))(M)$

5. Try to translate the following sentences into Type Theory.

   a. To wash yourself properly is important.
   b. It is healthy to love somebody
   c. To be perfect is to have all good properties.

6. Reduce the following expressions as much as possible by means of β-reduction.

a. $\lambda x(M(x))(j)$
b. $\lambda Y(Y(j))(M)$
c. $\lambda y \lambda Y(Y(x))(j)(M)$
d. $\lambda x \exists y(R(x)(y))(j)$
e. $\lambda x \exists y(R(x)(y))(y)$
f. $\lambda Y(Y(j))(\lambda x(M(x)))$
g. $\lambda Y \exists x(Y(x))(\lambda y(R(x)(y))$